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Comments on "General solutions for thermopiezoelectrics with various holes under thermal loading" [Int. J. Solids Struct. 37 (2000) 5561–5578]

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Qin (2000) recently presented a unified solution in closed-form for an infinite thermopiezoelectric plate with various holes under thermal loading, which is the extension of the author's previous work (Qin, 1999a,b; Qin and Mai, 1999; Qin et al. 1999; Qin, 1998a,b). It is well known that nearly all the exact solutions for the hole problems in anisotropic media were restricted to the case of an elliptic hole in the previous literature, and thus Qin's works are very interesting and remarkable. Here the writer wishes to give some comments on the paper of Qin (2000) in a fully collaborative spirit. Below we will use the same notation as in Qin (2000).

(1) The mapping function of Qin (2000) is not consistent with that of Hwu (1990). Qin (2000) studied a generalized two-dimensional problem of thermopiezoelectrics with various holes, the contour of which, say Γ, can be represented by

$$x_1(\psi) = a(\cos\psi + \gamma e_{m1}\cos m\psi), \qquad x_2(\psi) = a^e(\sin\psi - \gamma e_{m1}\sin m\psi) \tag{1}$$

Using following identities

$$\cos m\psi = \frac{1}{2}(\sigma^{-m} + \sigma^m), \qquad \sin m\psi = \frac{\mathrm{i}}{2}(\sigma^{-m} - \sigma^m)$$

where $\sigma = \cos\psi + \mathrm{i}\sin\psi$ $(0 \leqslant \psi \leqslant 2\pi)$, Eq. (1) can be rewritten as

$$\begin{aligned} x_1(\sigma) &= \frac{a}{2}[\sigma^{-1} + \sigma + \gamma e_{m1}(\sigma^{-m} + \sigma^m)], \\ x_2(\sigma) &= \frac{\mathrm{i}a}{2}[e(\sigma^{-1} - \sigma) - \gamma e_{m1}(\sigma^{-m} - \sigma^m)] \end{aligned} \tag{2}$$

Let the region outside Γ be S in the z-plane. Then, by an affine transformation $z_k = x_1 + p_k x_2$, S and Γ will be transformed into S_k and Γ_k, respectively, in the z_k-plane, where S_k are still infinite regions with holes Γ_k, but the contours of holes become

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$$z_k(\sigma) = x_1(\sigma) + p_k x_2(\sigma) \tag{3}$$

Substituting Eq. (2) into Eq. (3) yields

$$z_k(\sigma) = a(a'_{1k}\sigma + a'_{2k}\sigma^{-1} + e_{m1}a'_{3k}\sigma^m + e_{m1}a'_{4k}\sigma^{-m}) \tag{4}$$

where

$$a'_{1k} = \frac{1 - \mathrm{i}p_k e}{2}, \quad a'_{2k} = \frac{1 + \mathrm{i}p_k e}{2}, \quad a'_{3k} = \frac{\gamma(1 + \mathrm{i}p_k)}{2}, \quad a'_{4k} = \frac{\gamma(1 - \mathrm{i}p_k)}{2} \tag{5}$$

To seek k complex functions $f_k(z_k)$ which satisfy the given boundary conditions, it is convenient to introduce the mapping functions $z_k(\zeta_k)$ which conformally map S_k onto the exterior of a unit circle. To do this, Qin (2000) takes $z_k(\zeta_k)$ as

$$z_k(\zeta_k) = a(a_{1k}\zeta_k + a_{2k}\zeta_k^{-1} + e_{m1}a_{3k}\zeta_k^m + e_{m1}a_{4k}\zeta_k^{-m}) \tag{6}$$

in which

$$a_{1k} = \frac{1 - \mathrm{i}p_k e}{2}, \quad a_{2k} = \frac{1 + \mathrm{i}p_k e}{2}, \quad a_{3k} = \frac{\gamma(q + \mathrm{i}p_k e)}{2}, \quad a_{4k} = \frac{\gamma(1 - \mathrm{i}p_k e)}{2} \tag{7}$$

On the hole rim, $\zeta_k = \sigma$, and thus (6) produces

$$z_k(\sigma) = a(a_{1k}\sigma + a_{2k}\sigma^{-1} + e_{m1}a_{3k}\sigma^m + e_{m1}a_{4k}\sigma^{-m}) \tag{8}$$

Comparing Eq. (5) with Eq. (7), one finds that Eq. (8) is different from Eq. (4) in the general cases. This implies that the solutions based on Eq. (6) with (7) may not be the solutions of the hole described by Eq. (1).

(2) Qin's solution (2000) may lead to erroneous results at some points of the piezoelectric plate. As pointed out by Hwu (1990), it is necessary that all the roots of the equation $\partial z_k/\partial\zeta_k = 0$ be located inside the unit circle. However, in the actual analysis of Qin (2000), the necessary condition cannot be satisfied when $m > 1$. In this case, $\partial z_k/\partial\zeta_k = 0$ will have $m - 1$ roots outside the unit circle. This implies that at all the points of region S_k which correspond to these roots, $\partial\zeta_k/\partial z_k$ will lead to infinite values, and thus all the equations related to $\partial\zeta_k/\partial z_k$, for example, the equations of stress field and electric field (noting $f'_k(z_k) = f'_k(\zeta_k)/z'_k(\zeta_k)$), and Eq. (56) in Qin (2000), will produce erroneous results at the above points.

(3) On the other hand, Eq. (6) can be rewritten as

$$z_k = a\left[\omega(\zeta_k) + m_k\overline{\omega}\left(\frac{1}{\zeta_k}\right)\right], \quad (z_k \in S_k, |\zeta_k| \geqslant 1) \tag{9}$$

where

$$\omega(\zeta_k) = a_{1k}(\zeta_k + e_{m1}\gamma\zeta_k^{-m}), \qquad m_k = \frac{a_{2k}}{a_{1k}} \tag{10}$$

Eq. (9) has the same form as the mapping function used by Martynovich (1976), who addressed the two-dimensional problem of an anisotropic plate with a curvilinear hole. As pointed out by Kosmodamianskii, Lekhnitskii and Lozhkin (1979), Eq. (9) is incorrect.

Finally, the writer concludes that the mapping function (6) is not everywhere-conformal, and the use of it may lead to erroneous results. In addition, it should be noted that it is impossible to obtain the closed-form solutions to the problem of anisotropic media with an arbitrary shape hole. For a large class of such problems, only a numerical solution can be obtained by using the approximate methods of Lekhnitskii (1968) or Kosmodamianskii (1965).

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